



Research Article

Non-Uniform Relationship for Soil-Foundation Reaction

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Received: 26 June 2022; Revised: 31 July 2022; Accepted: 31 July 2022; Published: 31 July 2022

Abstract

This paper presents an exact formula for considering soil stiffness under the foundation. This formula derived from a 3D modeling of eight strip footings with different foundation widths supported on a soil medium. In the case of an earthquake, the behavior of the substructure soil plays an important role in the response of the structure. Studies show that the dynamic response spectrum of a structure on a flexible foundation is different from the response spectrum of a structure on a rigid foundation. Hence it is important to model the soil medium correctly to reach the best seismic results. There are different methods for modeling soil-structure interaction. These models use a constant value for modeling the soil stiffness. However, the soil stiffness varies along the foundation and should be calculated by analytical studies. This paper provides a simple formula which shows a non-uniform soil stiffness under the strip foundation and can be used for practical purposes.

Keywords:

Soil stiffness, Soil-structure interaction, Winkler method, Flexible foundation, Modeling soil-structure interaction

Cite this article as: Hajian M, Attarnejad R. (2022). Non-Uniform Relationship for Soil-Foundation Reaction. Civ Proj J, 4(5), 11–20. <https://doi.org/10.22034/cpj.2022.349082.1145>

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1. Introduction

Earthquake is a natural hazard that causes widespread damage, especially to the structural environment. Earthquakes is unpredictable in nature, and their characteristics are obtained according to possible considerations. In the case of an earthquake, the behavior of the substructure soil plays an important role in the response of the structure. In most cases, the soil medium is not modeled and its important effects are ignored. There are different methods for modeling soil-structure interaction. One of the well-known methods for considering the effects of soil-structure interaction is using Winkler model which present a constant value along the foundation. Design codes also uses the constant value to model soil stiffness in practical uses. Due to the simplicity of this method, this method cannot fully represent a continuous soil medium. The soil-foundation reaction varies along the foundation and it may be wrong to use a constant value (Kanwal, 2017; NIST, 2012; Kramer, 1996). The phenomenon of soil-structure interaction is mainly affected by the mechanism of energy exchanged between soil and structure. The importance of foundation interaction on the response of the structure depends on the characteristics of the soil system, foundation and structure and is strongly dependent on the characteristics of the foundation (Kanwal, 2017; Tabatabaiefar, 2012; Hosseinzadeh et. al., 2004; Torabi, and Rayhani, 2014; Nguyen, 2017; Ghannad, and Jahankhah, 2004; Mylonakis, and Gazetas, 2000; Kim and Roesset, 2004; Li et. al., 2014; Veletsos and Prasad, 1989; Hokmabadi et. al., 2014; Stewart et. al., 1998)

Over the past decades, many researchers have studied the phenomenon of soil-structure interaction and its effect on the response of different structures. In general, there are two types of methods used to model soil-structure interaction problems: (1) structural method (p-elastic beam or Winkler methods); (2) the continuous environment method, which is described below (Aron and Jonas, 2012; Teodoru, 2009). Tabatabaie far *et al.* (2013) investigated the effects of soil-structure interaction on medium-height moment-resisting frames on soft soil. They concluded that soil-structure interaction changes the structure response and so the conventional seismic design methods cannot guarantee the safety of medium-height moment-resisting frames on soft soil. Raheem *et al.* (2015) investigated the fundamental period of the structure by considering the soil-structure interaction. They observed that the fundamental period is higher when soil-structure interaction is considered. The effects of soil-structure interaction increase with the number of stories. The floor drift ratio increases with the number of stories. The lower the stiffness of the soil, the higher the stories drift of the stories. Hokmabadi *et al.* (2014) studied the effects of soil-pile-structure seismic interaction on the dynamic response of buildings. Torabi and Rayhani (2014) found that tall and slender structures are very sensitive to the effects of soil-structure interaction, including natural frequency change, foundation rocking motion and additional shear demand. Structure-to-dimension stiffness ratios are important parameters for controlling the performance of the foundation in structures with flexible foundations. Tahghighi and Rabiee (2015) also investigated the effects of soil-structure interaction on flexural frames. In this study, the performance of multistory systems was investigated by Winkler model method. In this study, they reached the following results. As the soil stiffness decreases, the response period of the fundamental period increases. The height of buildings has little effect on the period ratio, and this factor may not be considered for evaluating the properties of the system eigenvalue. The effective period of the soil-structure interaction system is longer than the period of the fixed base system due to rocking in the first mode. Nguyen (2017) has investigated the effect of foundation properties on the seismic performance of medium-height structures by considering the soil-foundation-structure interaction. For this purpose. Larger surface foundations can moderate lateral displacement amplitudes and consequently story drift caused by soil-foundation interaction. This can be cost effective in controlling the performance of our site. Ma *et. al.* (2009) have shown that the rocking motion of the foundation can be a critical mode of vibration of the foundation. Shallow foundations with different dimensions experience different amounts of rocking under a specific earthquake, and it is the rocking component that intensifies the lateral displacement of the upper structure and can affect the overall stability. Despite this, a considerable amount of seismic energy is wasted due to rocking motion, which is actually directly related to low shear forces to the upper structure. According to Gazetas and Milonakis (1998), the soil environment allows some movements due to its flexibility, which reduces the overall stiffness of the structure and thus increases the natural periods of the system. Chen *et al.* (2010) investigated the effects of foundation dimensions on the seismic response

of moment frames using the non-linear Winkler beam model. Mat foundations without embedment depth resulted in larger foundation deformations (slip and rocking motion).

Today, the most famous model used to model foundations for soil-structure interaction analysis by structural engineer is the Winkler model. Also, the Winkler (1867) model assumption is the oldest and simplest method for modeling the underlying environment of a structural system. In this method, the soil environment is modeled as a number of close, independent, linear and elastic springs on a rigid bed. There are a limited number of springs for a structural model (Hetényi, 1946; Coşkun, 2000, Wang et. al., 2005). The physical representation of Winkler method is shown in Figure 1.

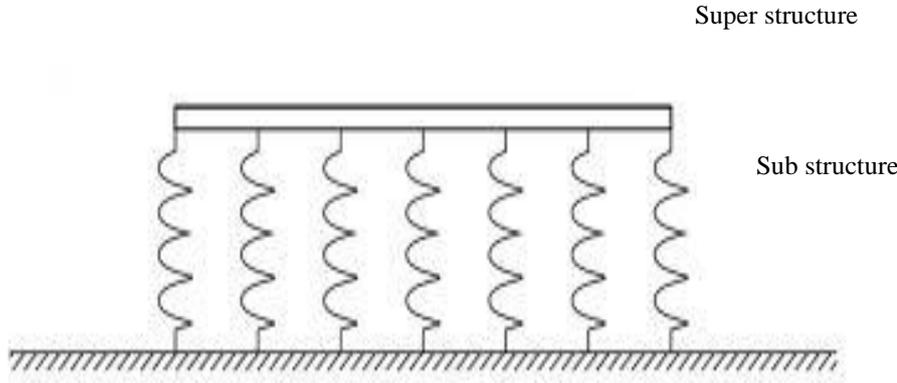


Fig. 1. Winkler method (Aron, and Jonas, 2012).

An important issue in the application of Winkler models is the calculation of the modulus of Winkler springs. For vertical loading, the modulus of springs are defined as follows. The individual spring stiffness is always defined as the relationship between the displacement and the reaction force. Thus, the modulus of the springs is written as Equation 1:

$$k(z) = \frac{p(z)}{W(z)} \quad (1)$$

Where $p(z)$ represent the vertical reaction of soil per unit length of pile and $W(z)$ is the corresponding settlement at depth z . $k(z)$ is the reaction modulus of substructure (soil reaction coefficient) and is expressed in terms of force per unit area. The purpose of this paper is to determine the best formula for $k(z)$ along the foundation length (Mylonakis, 2001). According to Gazetas (1991), static stiffness of rigid foundation at ground surface for two translational and one rotational degrees of freedom are considered as:

$$k_z = \frac{2G_s L}{1-\nu_s} \left[0.73 + 1.54 \left(\frac{B}{L} \right)^{0.75} \right] \quad (2)$$

$$k_x = \frac{2G_s L}{2-\nu_s} \left[2 + 2.5 \left(\frac{B}{L} \right)^{0.85} \right] - \frac{0.2}{0.75-\nu_s} G_s L \left(1 - \frac{B}{L} \right) \quad (3)$$

$$k_{yy} = \frac{G_s}{1-\nu_s} (I_y)^{0.75} \left[3 \left(\frac{L}{B} \right)^{0.15} \right] \quad (4)$$

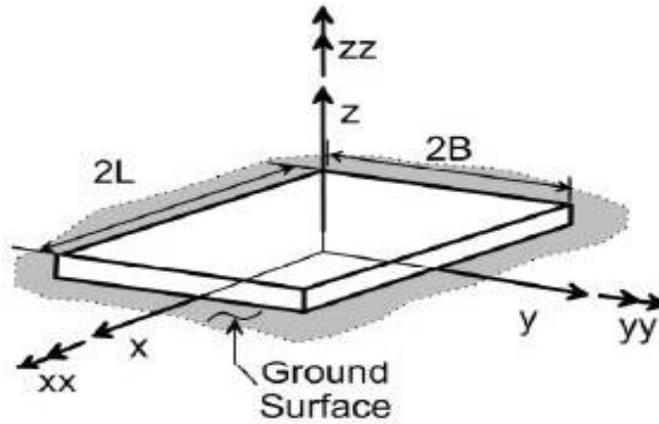


Fig. 2. Foundation's local axes (NIST, 2012).

where L and B are the length and the width of the foundation (see figure 2), respectively. G_s is the shear modulus (reduced for large strain effects), ν_s is the Poisson's ratio and I_y is the area moment of inertia of soil-foundation contact.

2. Finite Element Model

In this study, eight strip foundations of 1, 1.5, 2, 2.5, 3, 3.5, 4, 4.5 m widths with the length of 12 m have been modeled in OPENSEES finite element software programming. The foundations rested on a 3D soil medium using SSquad finite elements. The soil depth was assumed to be 30 m, and the width was assumed to be 7 times the length of the foundation. The properties of the soil are as follows: a shear modulus of $33.1 \cdot 10^7$ N/m, a poisson ratio of 0.4 and a mass density of 1470 kg/m³. Each foundation was analyzed and the results are shown in section 3. For comparison, the Gazetas vertical elastic stiffness of the foundation should be calculated. As an example, the total vertical elastic stiffness for foundation 1×12 m is:

$$k_{Gazetas} = k_z = 664552029.2 \text{ N/m}^2$$

3. Foundation Results

To obtain the stiffness of the soil-foundation interface, the reaction forces and the displacements of the foundation sub elements were calculated. Tables 1 and 2 indicates the force reaction and displacement outputs foundations, respectively. Since the foundation reaction diagram is symmetric about the center axis of the beam, the outputs were only shown for one side of the beam. As shown in figure 3, each foundation was divided into 12 sub-elements, and the results were shown for each joint. By using joint forces and displacements, soil-foundation stiffness calculated, as shown in table 3. Furthermore, Force reactions, displacements and soil stiffness are shown in figures 2, 3 and 4 respectively.

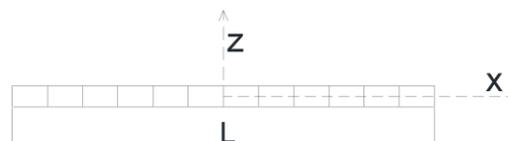


Fig. 3. Foundation's sub-elements.

Table 1. Force reaction of the foundation joints.

B (m)	joint forces												
1	15130	11746	12824	12313	15807	17728	18977	19748	20072	19836	18719	15404	16933
1.5	21097	15910	16753	15697	19957	22226	23664	24505	24782	24361	22864	18724	20537
2	27001	20039	20658	19058	24075	26684	28300	29203	29431	28830	26966	22018	24119
2.5	32881	24151	24541	22396	28160	31100	32887	33846	34024	33245	31022	25278	27665
3	38750	28253	28408	25711	32214	35477	37429	38441	38566	37611	35032	28503	31174
3.5	44617	32349	32261	29006	36238	39819	41929	42990	43060	41929	38999	31693	34646
4	50484	36440	36101	32283	40236	44127	46392	47497	47511	46204	42924	34850	38082
4.5	56353	40529	39931	35544	44210	48406	50819	51965	51920	50438	46812	37976	41484

Table 2. Displacements of the foundation joints.

B (m)	joint displacements $\times 10^{-4}$												
1	5.35	5.73	6.08	6.40	6.70	6.98	7.23	7.46	7.66	7.83	7.97	8.07	8.10
1.5	7.14	7.55	7.93	8.29	8.62	8.92	9.20	9.46	9.68	9.87	10.02	10.11	10.15
2	8.93	9.36	9.77	10.16	10.52	10.86	11.17	11.44	11.68	11.89	12.05	12.15	12.19
2.5	10.70	11.17	11.61	12.03	12.42	12.78	13.12	13.42	13.68	13.90	14.07	14.17	14.21
3	12.48	12.97	13.44	13.89	14.31	14.70	15.06	15.38	15.66	15.89	16.07	16.19	16.23
3.5	14.25	14.77	15.27	15.75	16.19	16.61	16.99	17.33	17.63	17.88	18.07	18.19	18.23
4	16.02	16.57	17.09	17.60	18.07	18.51	18.91	19.28	19.59	19.85	20.05	20.17	20.22
4.5	17.80	18.36	18.92	19.44	19.94	20.41	20.83	21.21	21.54	21.81	22.02	22.15	22.20

Table 3. Stiffness of the foundation joints.

B (m)	joint stiffness $\times 10^7$												
1	4.559	4.345	4.275	4.296	4.315	4.323	4.321	4.311	4.297	4.284	4.272	4.265	4.262
1.5	4.882	4.504	4.332	4.301	4.295	4.292	4.287	4.280	4.271	4.262	4.255	4.251	4.249
2	5.062	4.601	4.371	4.309	4.286	4.275	4.267	4.259	4.252	4.246	4.240	4.237	4.236
2.5	5.179	4.667	4.400	4.317	4.282	4.264	4.253	4.245	4.238	4.232	4.228	4.225	4.224
3	5.262	4.715	4.422	4.325	4.280	4.257	4.243	4.233	4.226	4.221	4.217	4.214	4.213
3.5	5.326	4.753	4.440	4.331	4.279	4.251	4.235	4.224	4.216	4.211	4.207	4.205	4.204
4	5.376	4.783	4.456	4.337	4.279	4.247	4.229	4.216	4.208	4.202	4.198	4.196	4.195
4.5	5.416	4.809	4.468	4.342	4.279	4.244	4.223	4.210	4.201	4.195	4.191	4.189	4.188

The results of Table 3 are used to obtain the best relationship which can model the soil stiffness under the foundation. First, we calculate the relationship of the stiffness of the subsoil for foundation of $B = 1$, and then we obtain the relationship of the other foundations compared to $B = 1$. Figure 4 shows the computational stiffness relationship for a foundation with $B = 1$ which is considered as equation 5.

$$\frac{K(x)|_{(B=1)}}{area} = 8 \times 10^{-7} x^6 + 6 \times 10^{-6} x^5 - 3 \times 10^{-5} x^4 + 2 \times 10^{-13} x^3 + 0.0004 x^2 - 7 \times 10^{-12} x + 0.082 \quad (5)$$

Area is the parameter correspond to the area under the curve, which can be calculated by $K_{Gazetas}$ as equations 6 and 7 (see figure 5):

$$area = K_{Gazetas} \times \alpha \quad (6)$$

$$\alpha = \frac{2.615}{B^{0.73}} \tag{7}$$

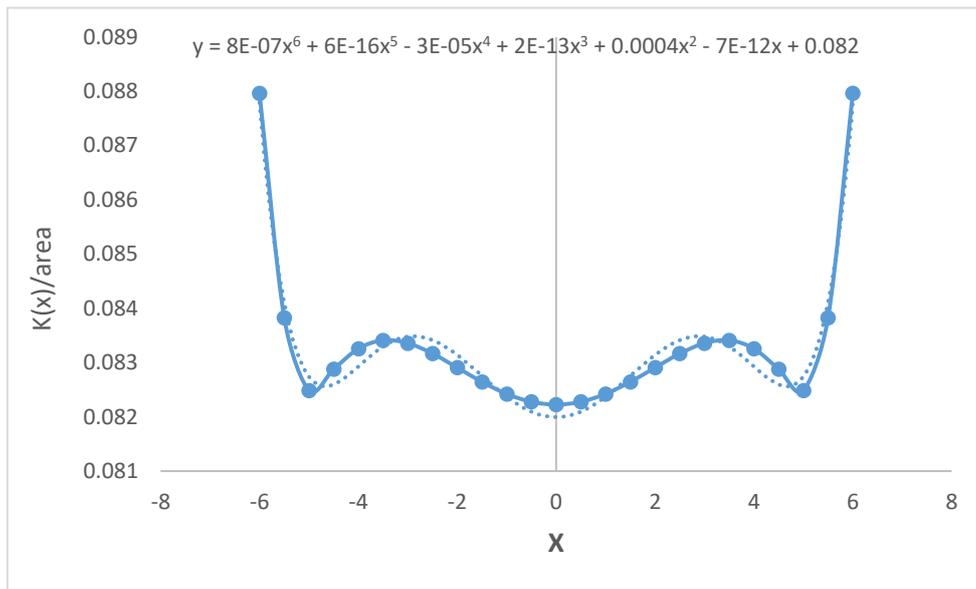


Fig. 4. The soil stiffness under the foundation.

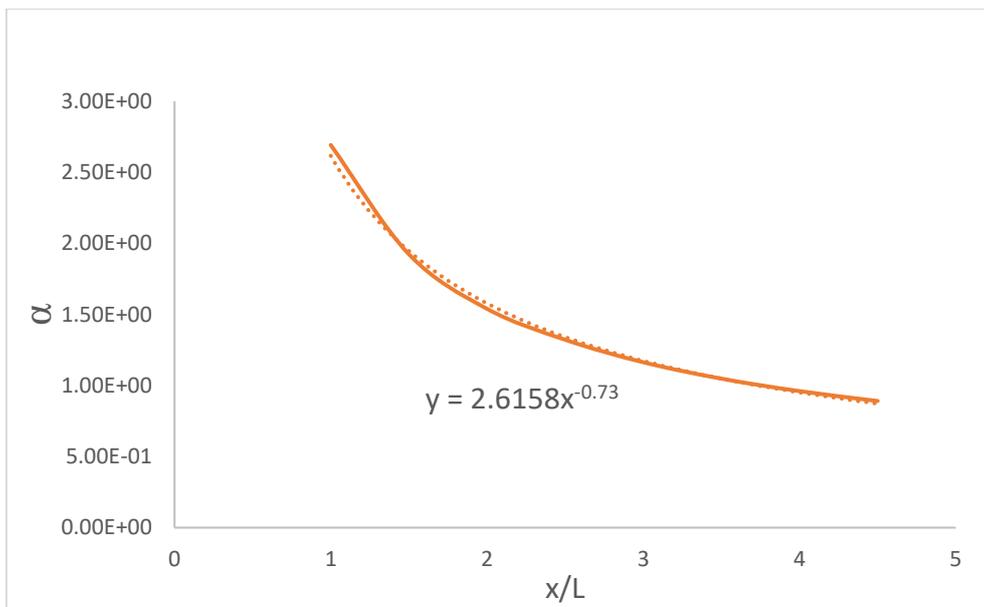


Fig. 5. The relationship between $K_{Gazetas}$ and area under the curve.

The above relationship is not a practical formula for using in design codes, so it should be simplified. The simplified model of equation 5 is as equation 8. Note that the vertical soil reaction is not uniform, and tends to increase near the foundation edges (of length $R_e L$). According to NIST (2012), to correct for underestimation of rotational stiffness, strips along the foundation edge are assigned stiffer springs (R_e is typically in the range of 0.3 to 0.5). Figures 7, 8 and 9 show the relationships for equations 8 and 9.

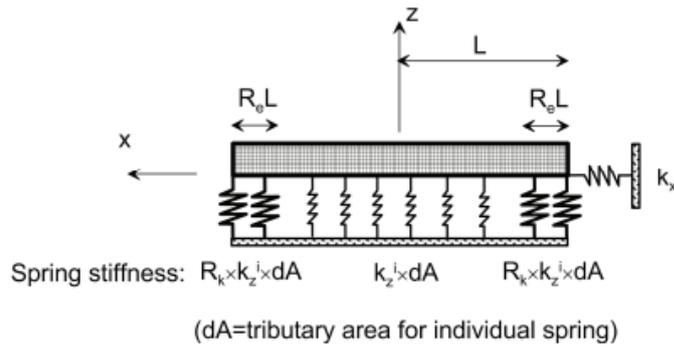


Fig. 6. Vertical spring distribution according to NIST (2012).

$$\frac{K(x)|(B=1)}{area} = \begin{cases} 0.0993(x/L) + 0.0383 & \text{for } ReL \\ -0.1022(x/L)^3 + 0.0703(x/L)^2 - 0.01(x/L) + 0.0826 & \text{for other parts} \end{cases} \quad (8)$$

$$K(x) = K(x)|(B=1) \times \begin{cases} \frac{1}{B^{0.94}} \\ \frac{0.98}{B^{1.05}} \end{cases} \quad (9)$$

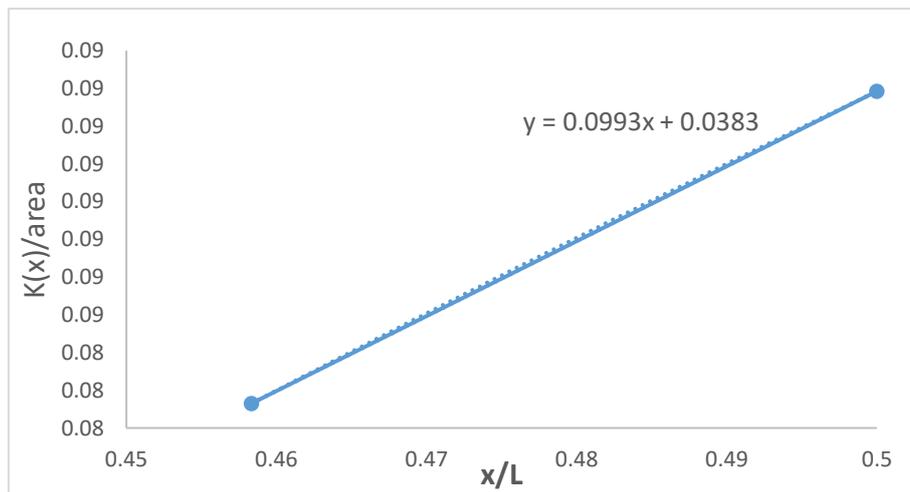


Fig. 7. The simplified soil stiffness (part 1).

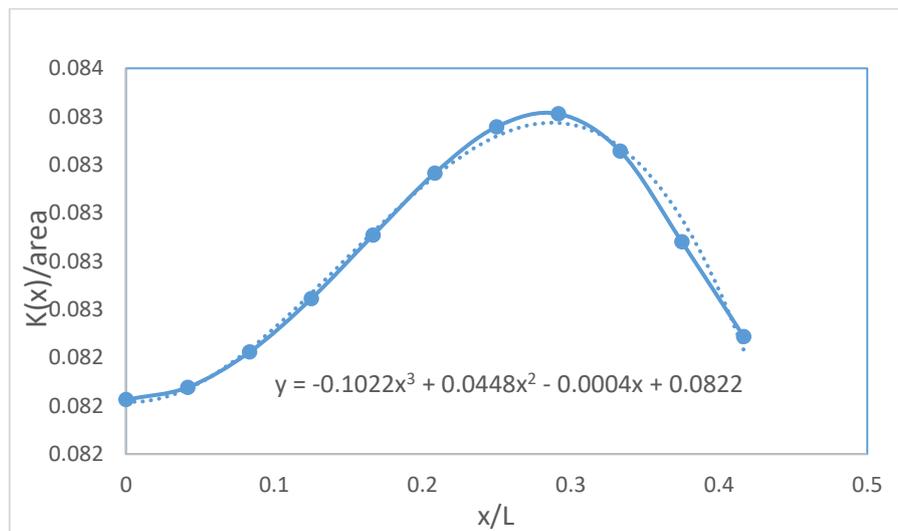


Fig. 8. The simplified soil stiffness (part 2).

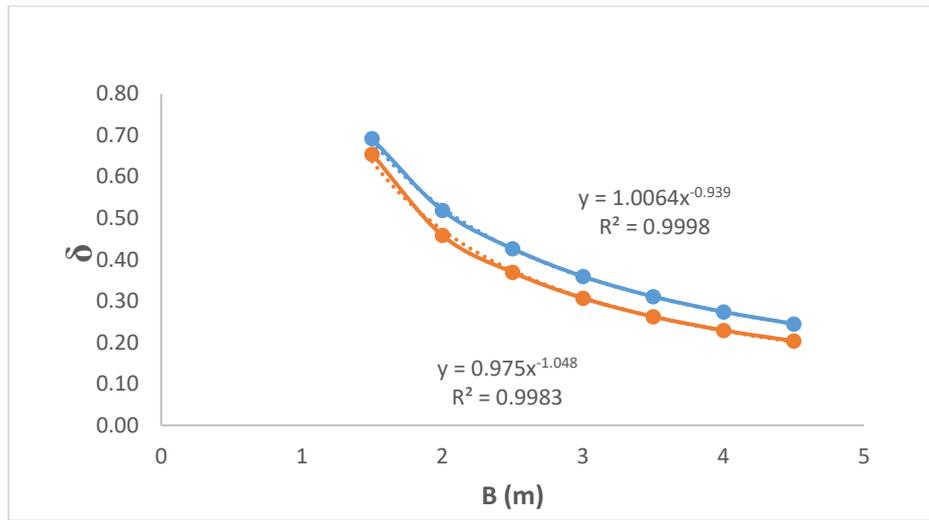


Fig. 9. The relationship between $K(x)$ and $K(x) I_{(B=1)}$.

The increase in spring stiffness at foundation edge can be calculated as equations 12 and 13 (see figure 10). R_{k-code} is the increase stiffness of Nist (2012) and can be calculated as a function of foundation end length ratio, Re .

$$k_{max} = k_{average} \times R_k \quad (10)$$

$$R_k = R_{k-code} \times (0.0214B^3 - 0.259B^2 + 1.053B - 0.394) \quad (11)$$

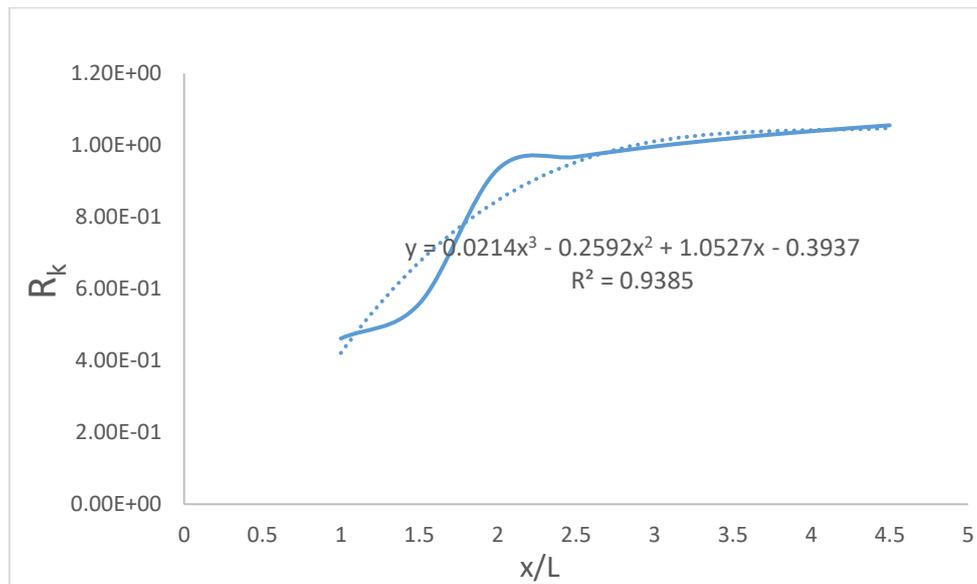


Fig. 10. The increase in stiffness parameter, R_k .

4. Conclusion

The soil-structure interaction has an important effect on the seismic response of structures. Soil flexibility can change the dynamic response of high-rise structures, especially when the structure is placed on soft soil. Soil dynamic parameters are significantly affected by substrate flexibility. Therefore, it is necessary to provide an exact method for modeling soil structure interaction and their effects. A simple method for modeling a flexible bed is to use Winkler springs. In this paper, the soil reaction of eight strip foundations with different widths were investigated by OPENSEES software in 3D manner. Then the best relationship for stiffness ratio soil under the foundation was formulated to use in practical applications.

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