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Research Article

Coupling Effect and Superposition Law in Soil Treatment Systems Incorporating Vacuum Preloading

Mohammad Mehdi Pardsouie ^{۱*}, Mehdi Mokhberi ^۲, Seyed Mohammad Ali Zomorodian ^۳, Mohammad Hadi Pardsouie ^۴

۱. PhD candidate, Pars Geometry Consultants, Tehran, Iran

۲. Associate professor, Department of geotechnical engineering, Estahban Branch, Islamic Azad University, Estahban, Iran

۳. Associate professor, Department of Water Engineering, Faculty of Agriculture, Shiraz University, Shiraz, Iran

۴. Msc student, Department of civil engineering, Estahban Branch, Islamic Azad University, Estahban, Iran

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Abstract

In previous published literatures it was stated that superposition law might be valid for ground improvement techniques consisting of prefabricated vertical drains (PVDs) along with surcharge and vacuum preloading. Even some professional geotechnical engineers might think of this false idea that superposition law might be valid in such ground improvement techniques. It was shown that the superposition law is not valid because of the hydro-mechanical coupling interactions which exist between vacuum and surcharge preloading. A case history was presented and Finite Element Modeling (FEM) was used for verification and the demonstration of coupled consolidation interaction between vacuum and surcharge preloading. Three main parameters as settlement, lateral displacement, and water excess pore pressure were evaluated for different scenarios. The results that are based on a macro-element approach can be used for better comprehension of the working mechanism of combined treatment systems. Considering the results of this literature, a complex combined vacuum and surcharge preloading can be broken in simpler cases that can be used for either deriving analytical or empirical solutions.

Keywords: coupling, superposition, vacuum preloading, finite element modeling, excess pore pressure

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*Corresponding author E-mail address: m.m.pardsouie@gmail.com

۱. Introduction

Vacuum consolidation is a technique that is used along with PVDs and surcharge preloading to accelerate the process of consolidation of weak clay or peat soils and meanwhile reduce the issues with ground heaves in the premier of the embankment. Based on different soil conditions or project working speculations, different systems or design might be considered (D. T. Bergado et al., ۲۰۲۲; Wu et al., ۲۰۲۲; R.-J. Zhang, Zheng, Dong, & Zheng, ۲۰۲۲). FEM is a common tool that is used extensively by consultants to model the soil behavior before, during and after the reclamation process (Pardsouie, Pardsouie, Zomorodian, Mokhberi, & Application, ۲۰۲۲; R.-J. Zhang et al., ۲۰۲۲). The super structures under construction, soil layer specifications, and machinery availability are the main parameters that determine the final design parameters including sealing measures (Anda et al., ۲۰۲۰; Long, Nguyen, Bergado, Balasubramaniam, & Geomembranes, ۲۰۱۵), PVD depth (Griffin & O'Kelly, ۲۰۱۴; Long et al., ۲۰۱۵) and spacing (Long, Bergado, Nguyen, Balasubramaniam, & AGSSEA, ۲۰۱۳; Wang, Yu, Zhou, & Wang, ۲۰۲۰), and the required preloading pressure (Bhosle & Deshmukh, ۲۰۱۸; de Lillis, Fasano, Flora, & Miliziano, ۲۰۲۲). (Mesri & Khan, ۲۰۱۲) State that there is no difference in the magnitude and rate of settlement resulting from a vacuum load and an equivalent fill load. Settlement analysis for vacuum or vacuum plus fill loading can be carried out using the procedures that are available for fill loading. (JC Chai, Carter, Hayashi, & engineering, ۲۰۰۵) Assuming that the volumetric strain due to vacuum consolidation is the same as for 1D consolidation with a surcharge load of the same magnitude, proposed an approximate method for calculating the ground settlement and inward lateral displacement induced by vacuum preloading. (Jinchun Chai, Ong, Carter, & Bergado, ۲۰۱۳; Wang et al., ۲۰۱۹) proposed an empirical equation for the estimation of lateral displacement. In their solution, they proposed that vacuum pressure induces negative pore pressure while embankment loading induces positive pore pressure. (Pardsouie & Pardsouie, ۲۰۲۲) performed an investigation on the effect of PVDs length on the magnitude of lateral displacement.

(Flessati, Di Prisco, & Callea, ۲۰۲۱) stated that the macro-element approach nowadays is largely considered to be a successful theoretical tool for solving soil-structure interaction problems. This approach is based on the definition of a generalized constitutive law putting in relation a small number of suitably defined generalized stress/strain variables and can be used as designing tool according to ultimate limit state and displacement based approaches. Particularly in the last decades, the application of the macro-element approach in soil-structure interaction problems has gained an increased popularity in the practical and academic implications (Flessati et al., ۲۰۲۱; Vlahos, Cassidy, & Martin, ۲۰۱۱; Y. Zhang, Cassidy, & Bienen, ۲۰۱۴).

In this literature, the governing equation for combined vacuum and surcharge preloading is explained and then a case history is introduced and verified based on existing data and then the model is discretized and investigated for different conditions and scenarios to illuminate some misunderstandings or false ideas regarding the combined system of preloading for ground improvement, especially the explanation of negative excess pore pressure, superposition law validness, lateral displacement on surface ground due to the vacuum preloading and the effect of hydro-mechanical coupling.

۲. Governing equation of combined vacuum and surcharge preloading in a 1D condition

(Mohamedelhassan & Shang, ۲۰۰۲) conducted some laboratory tests on different clay specimens under surcharge and vacuum preloading. An analytical solution for the prediction of excess pore pressure was proposed assuming Terzaghi 1D small strain and also superposition law. The equations are as follows:

$$\frac{\partial u}{\partial t} = c_{vs} \frac{\partial^2 u}{\partial z^2} \quad (0 < z < H, t > 0) \quad (1)$$

$$\frac{\partial u}{\partial t} = c_{vv} \frac{\partial^2 u}{\partial z^2} \quad (0 < z < H, t > 0) \quad (2)$$

Where t is the time, z is the depth, H is the drainage path, c_{vs} is the coefficient of consolidation for surcharge preloading, and c_{vv} is the coefficient of consolidation for vacuum preloading and u is excess pore water pressure. Assuming the validness of superposition law and stated initial and boundary conditions the equations were summed as:

$$u(z; t) = u_v(z; t) + u_s(z; t)$$

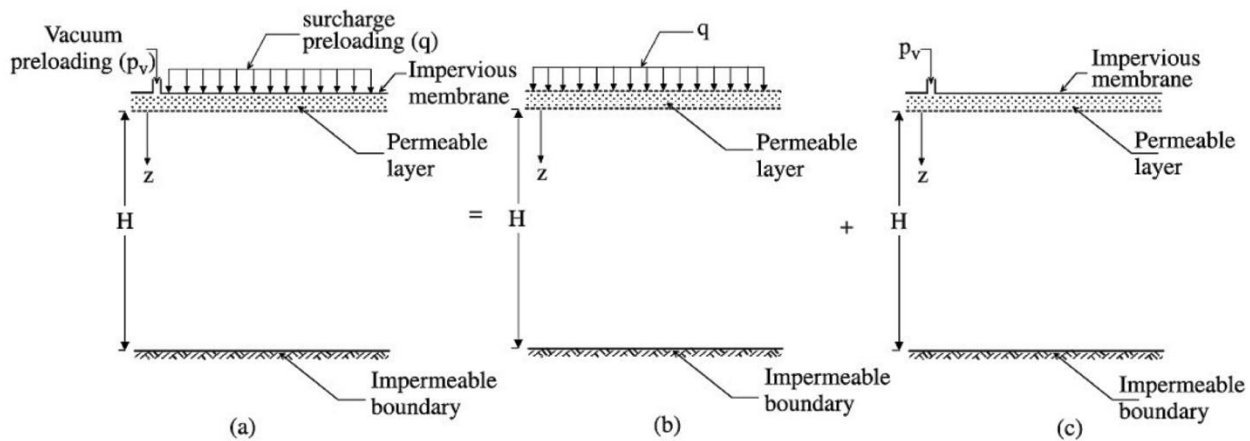


Figure 1: Schematic of (a) vacuum and surcharge combined preloading; (b) surcharge preloading; and (c) vacuum preloading (Mohamedelhassan & Shang, 2002)

Where u_v and u_s are excess pore pressure for vacuum and surcharge preloading respectively. Figure 1 shows the schematic of the equations of combined vacuum and surcharge for a surcharge preloading, q , and a vacuum preloading p_v .

(Gibson, Schiffman, & Whitman, 1989) defined the excess pore water pressure in the consolidation process in two ways as: excess over the hydrostatic pressure (the pressure distribution when the pore water is stationary) and the pore pressure in excess of a steady-state flow condition. These definitions explain best the two distinct mechanisms which govern in a combined vacuum and surcharge preloading. Applying surcharge preloading would increase the excess pore pressure and because of the very low permeability of clays it can't be dissipated which is under the first definition. On the other hand, applying vacuum pressure through PVDs creates a water head between soil and PVD which accelerates the flow in clay soils and eases the discharge of water which is in accordance with the second definition. Although the effect of vacuum preloading is somehow the same as surcharge preloading in the acceleration of the consolidation process, they shouldn't be mistaken with each other as they have two different mechanisms. The vacuum pressure effect is often demonstrated by negative

pore pressure. Care should be taken to not mistake the negative algebraic term with its real mechanism as it is the pore pressure in excess of a steady-state flow condition in soil in the vicinity of PVDs under vacuum preloading. (Lu, Likos, Luo, Oh, & Engineering, ۲۰۲۱) has discussed in detail the inefficiency of common definition of pore pressure in soil mechanics and emphasized the necessity for developing better theories and seeking better engineering solutions for problems in geotechnical engineering.

To clarify the difference, assume that at a given soil depth of z under combined vacuum and surcharge treatment system (+P) is the quantity of the excess pore pressure as a result of embankment surcharge and (-P) is the quantity of the excess pore pressure as a result of the vacuum pump. Assume the superposition law is valid and as result, there should be no settlement because of consolidation as the quantity of excess pore pressure is equal to zero i.e. (+P - P) but in contrast, the consolidation settlement would occur. Of course, this is not the case and it is an ideal situation in the real-world where considerable settlement takes place. It shows the superficial way of using superposition law. But now the question arises about if the absolute value of ($|-P|$) adopted in the analysis would be considering the case of (ψP) a reasonable approach in dealing with such a situation. This example demonstrates the actual performance of two distinct mechanisms of surcharge and vacuum preloading. In reality, none of the stated situations would occur. As it would be seen the superposition law is not valid and moreover, another phenomena exists which is the interaction between PVDs and vacuum and surcharge or the hydro-mechanical coupling (in brief coupling). In coupled consolidation analysis, the excess pore pressure and deformations would be calculated simultaneously while considering compressibility of soil particles and pore fluid (Biot, ۱۹۴۱) to observe the stated coupling effect where it can be increasing or decreasing based on different situations.

(Mohamedelhassan & Shang, ۲۰۰۲) reported a minor disagreement between analytical solutions and consolidation results which might be attributed to laboratory deficiencies or the coupling effect of vacuum and surcharge. Refer to formulas (۱), (۲), and (۳) and assuming constant permeability under various preloading (not a valid assumption) the simplified 1D excess pore pressure might be written as:

$$u(z; t) = u_v(z; t) + u_s(z; t) + u_{vs}(z; t) \quad (۴)$$

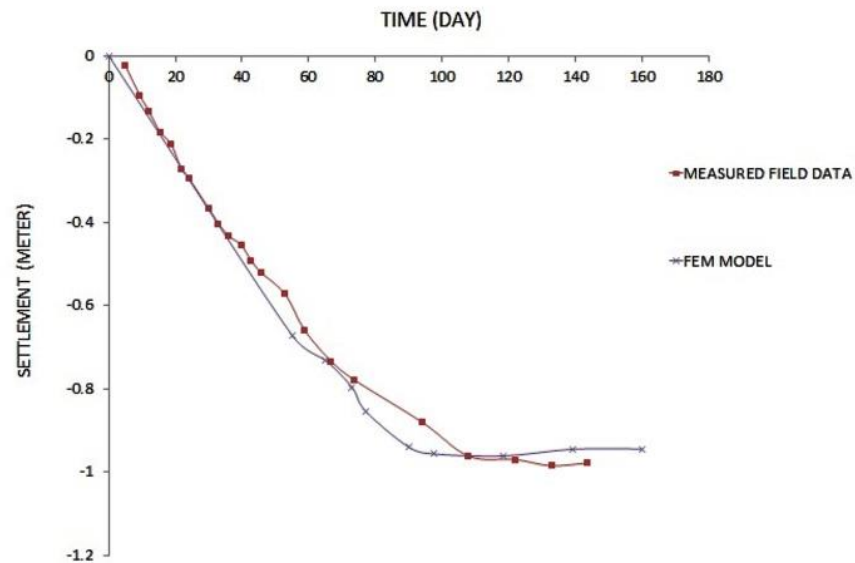
Where u_{vs} is the coefficient of consolidation for a combined surcharge and vacuum preloading that considers the hydro-mechanical coupling effect in analytical solutions.

۳. Field case history

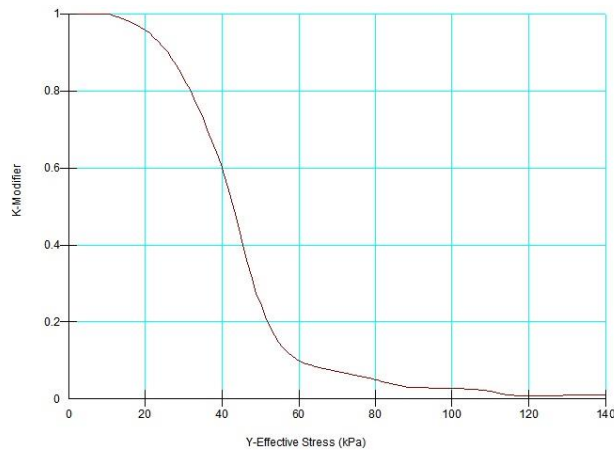
۳.۱. Model verification

(D. Bergado, Chai, Miura, & Balasubramaniam, ۱۹۹۸) has reported the monitoring results of two trial embankments in Second Bangkok International Airport and (Indraratna & Rujikiatkamjorn, ۲۰۰۶) modeled these two embankments using FEM modeling in plane-strain condition. The second trial embankment is used for primary model verification. This case history was specifically selected because of variable vacuum pressure that was applied. The related data concerning the history of preloading and material properties can be accessed through these articles. GEOSTUDIO ۲۰۱۸ SIGMA/W coupled analysis in plane-strain condition was used for modeling. It should be noted that the effect of well resistance and clogging were considered in the model by boundary conditions and the smear effect was considered by the approach proposed by (Indraratna & Redana, ۲۰۰۰). As stated by (Cai, ۲۰۲۱) in order to consider nonlinearity of the consolidation arising from evolving permeability and compressibility of the soil due to change in void ratio during consolidation and non-Darcian flow

regime for low permeability soil and large strain elasto-plastic behavior of the soil, a permeability modifier was applied in FEM analyses (geostudio, ۲۰۱۸).



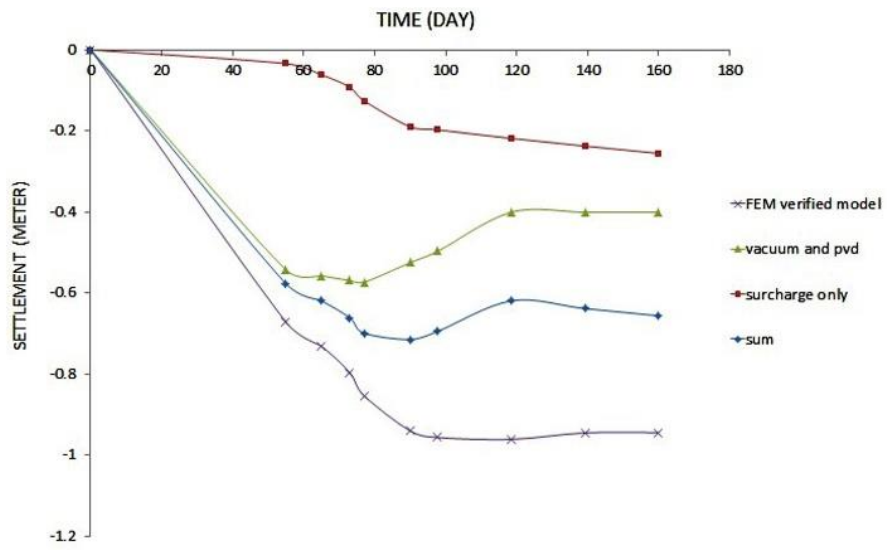
(a)



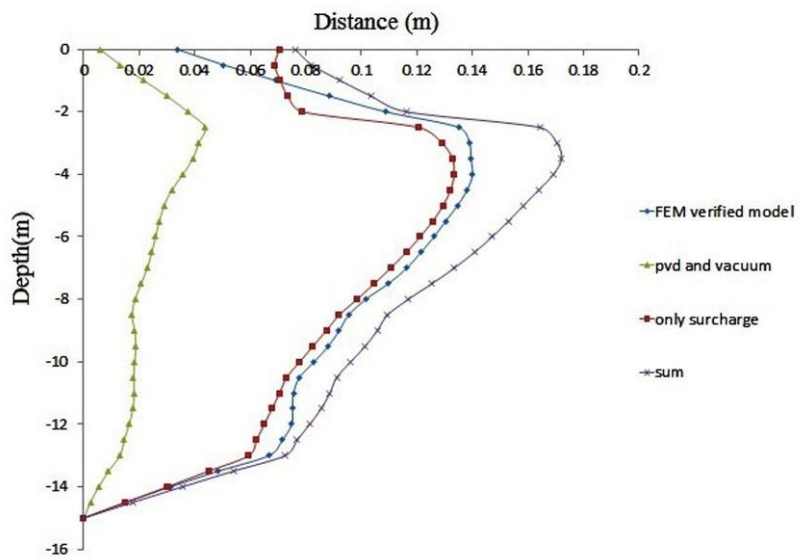
(b)

Figure ۲: (a) Verification of the second trial embankment Thailand international airport site (b) Hydraulic permeability modifier

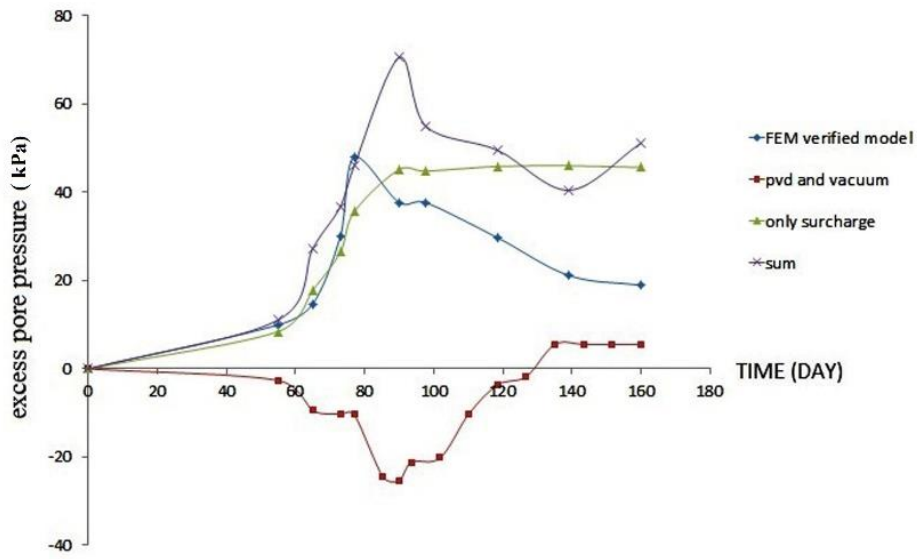
As reported by (D. Bergado et al., ۱۹۹۸) because of possible disturbances of the inclinometer casings near the ground surface, the lateral displacement data was not valid and the simulations didn't agree. For excess pore pressure because of the insufficiency of installed piezometer acquired data underestimated.



(a)

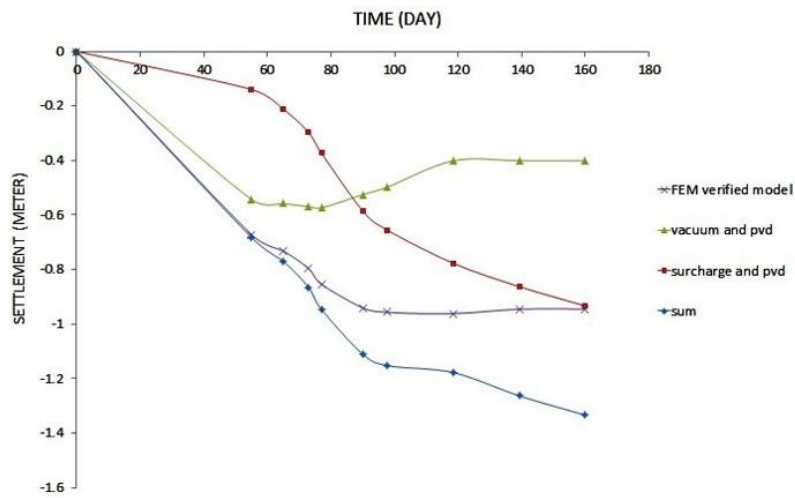


(b)

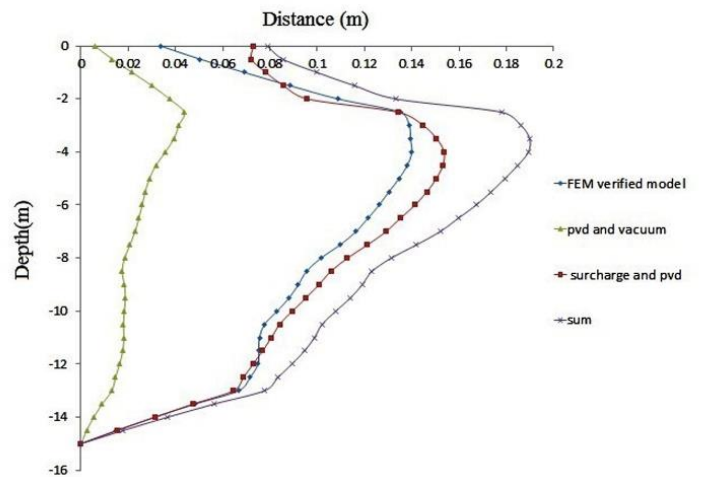


(c)

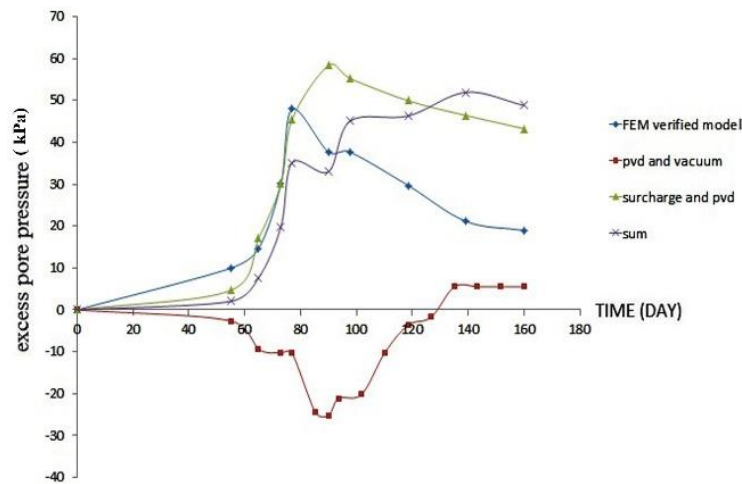
Figure ۳: Comparison of FEM simulations of (a) settlement curve (b) lateral displacement (c) excess pore water pressure for the verified FEM model vs. case('a) + case('b) (only surcharge + vacuum and PVDs) scenario



(a)



(b)



(c)

Figure 4: Comparison of FEM simulations of (a) settlement curve (b) lateral displacement (c) excess pore water pressure for the verified FEM model vs. case(1a) + case(1b) (surcharge and PVD + vacuum and PVDs) scenario

3.2. Superposition law and coupling in verified model

Based on the verified model two separate cases were considered as:

1. (a) Only surcharge preloading without PVDs and (b) vacuum and PVDs.
2. (a) surcharge and PVDs and (b) vacuum and PVDs

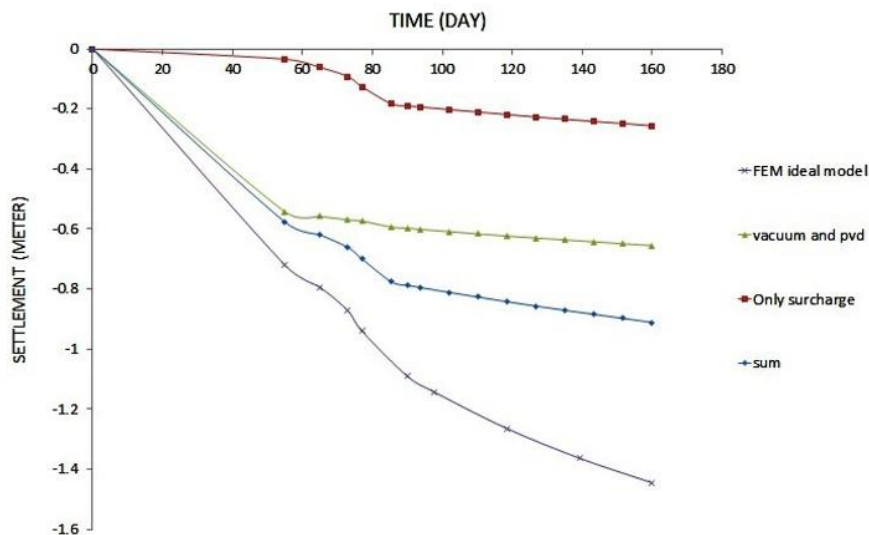
Since the percentage of PVDs contribution in consolidation is not clear both cases as in present and in absence of PVDs were considered. Three main parameters as settlement (centerline), lateral displacement (toe of the embankment), and excess pore pressure (3 meters beneath centerline) were evaluated. Figure 3 shows the results of FEM modeling where verified FEM models are shown vs. only surcharge, vacuum and PVDs, and the algebraic summation of only surcharge and vacuum and PVDs. It should be noted that in summation of all cases for excess pore pressure the absolute value of excess

pore pressure is considered. Although it is not wise at all, to sum up the excess pore pressure resulting from surcharge with resultant excess pore pressure of vacuum water head, unfortunately, there were no other ways to show their effect. For a demonstration of the effect of each case in the SUM value, the related individual curves are shown beside the SUM curve.

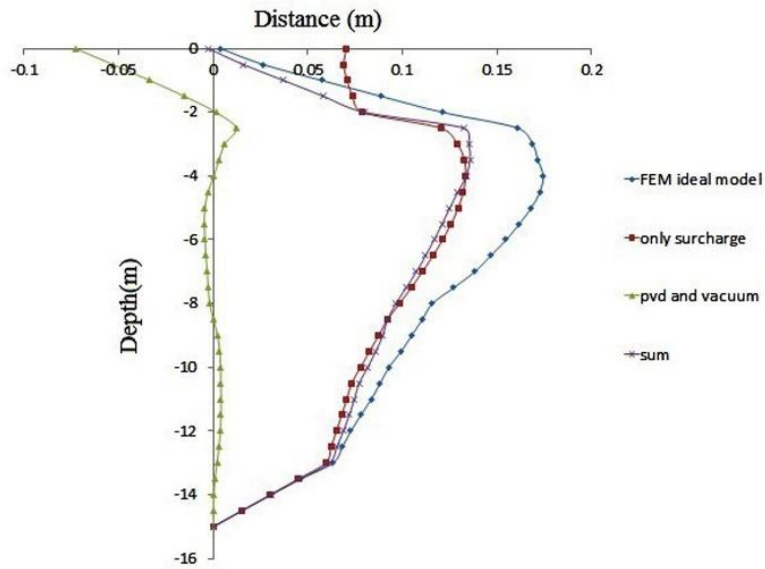
۳.۳. Superposition law and coupling in the ideal model

For better evaluation of superposition law validity and the coupling effect, two different models were simulated based on the verified model considering a constant ideal vacuum pressure of ۶۰ kpa assumption for all the ۱۶۰ days. Again two separate cases and three main parameters as settlement (centerline), lateral displacement (toe of the embankment), and the excess pore pressure (\bar{u} meters beneath centerline) was considered as:

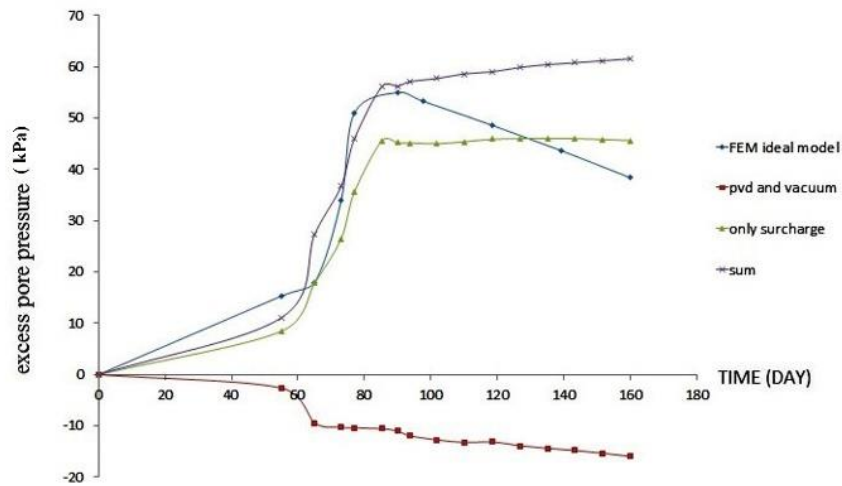
- ۳. (a) Only surcharge preloading without PVDs and (b) vacuum and PVDs.
- ۴. (a) surcharge and PVDs and (b) vacuum and PVDs



(a)



(b)

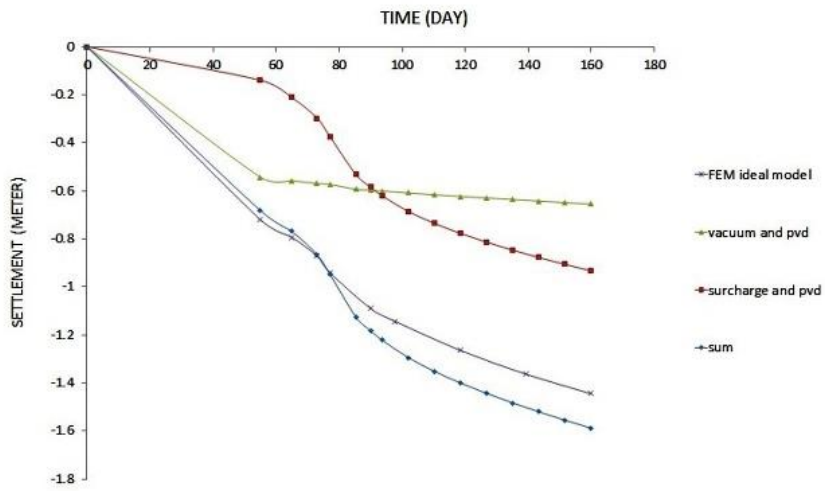


(c)

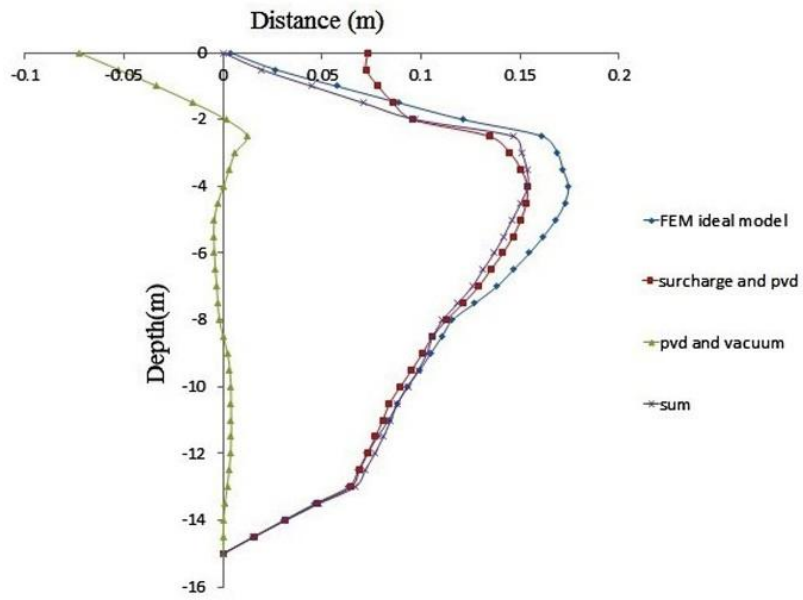
Figure ۵: Comparison of FEM simulations of (a) settlement curve (b) lateral displacement (c) excess pore water pressure for the ideal FEM model vs. case(۳a) + case(۳b) (only surcharge + vacuum and PVDs) scenario

۴. Discussion and Results

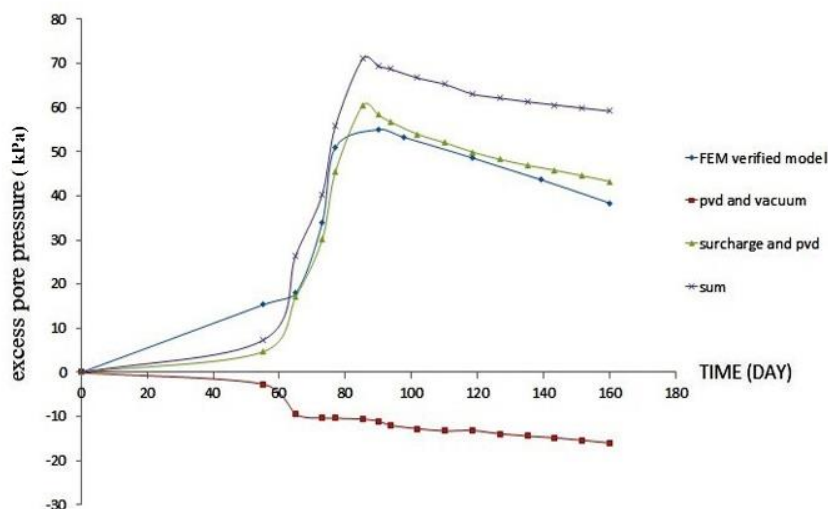
As it can be seen from all the simulations, the superposition law is not valid in proposed cases. The algebraic summation of cases based on different situations vs. FEM models is either decreasing or increasing. The interaction between PVDs, surcharge and the vacuum or the coupling effect can be easily seen on plotted curves.



(a)



(b)



(c)

Figure 6: Comparison of FEM simulations of (a) settlement curve (b) lateral displacement (c) excess pore water pressure for the ideal FEM model vs. case(ξa) + case(ξb) (surcharge and PVD + vacuum and PVDs) scenario

The predicted surface settlement curves from FEM (verified and ideal) models are compared with various scenarios. Figure 3(a) shows that in the absence of PVDs in only the surcharge model (case 1(a)) the results of the SUM case for settlement are underestimated after the 60th day. In contrast, by the inclusion of PVDs, the results as shown in figure 4(a) are overestimated after the 60th day. This is the time when the coupling effect starts. From the figure 4(a) the decreasing effect of coupling can be seen when the applied vacuum pressure is not constant and the ultimate settlement is lesser than the summation of the described cases. For the ideal constant 60 kpa vacuum case as shown in figure 5a the resultant SUM case curve is underestimated similar to figure 3(a). In the case with the inclusion of PVDs (6a) the results were overestimated only 9 percent after the 90th day. Figure 6(a) and 4(a) show that for discretizing complex models like combined surcharge and vacuum the inclusion of PVDs with surcharge gives better predictions for both cases with constant and variable applied vacuum pressure but for variable vacuum the results would be underestimated by 40 percent in the final settlement curve. By applying constant vacuum pressure the coupling effect has been minimized in settlement curves. Models with the inclusion of PVDs might be considered in the case of constant vacuum pressure for preliminary prediction or empirical equations for surface settlement (case 4(a) + case 4(b)). For variable vacuum pressure none of the models could predict the settlement.

The predicted lateral displacement curves from FEM (verified and ideal) models are compared with various scenarios. Figure 3(b) and 4(b) show that for the field variable applied vacuum the resultant curve for lateral displacement in the SUM case is overestimated while as shown in figure 5b and 6b the resultant curve for lateral displacement in the SUM case is underestimated for the ideal case. For the variable applied vacuum in the field the predicted lateral displacement from the SUM case is twice as compared to the FEM model at ground surface. It can be seen that for this case the coupling effect reduced the lateral displacement at the ground surface from 6cm to 4cm and reduced the lateral displacement by about 30 percent in the very soft clay layer under the surface. For cases with variable vacuum pressure considering the lateral displacement from the case surcharge without PVDs (case 1(a)) might be considered for empirical equation except for ground surface where 90 percent of the SUM cases (case 1(a) + case 1(b) or case 3(a) + case 3(b)) in both cases of surcharge with and without PVDs might be considered for preliminary prediction or empirical equations related to lateral

displacement. In contrast, for the ideal constant ۶۰ kpa case, the SUM case is underestimated below the surface but agrees well on the ground surface. For constant vacuum pressure, the coupling effect would increase the lateral displacements for weak clay layers under the surface but don't affect the ground surface that might be attributed to over-consolidation of the surface layer. Both models of the SUM cases with the inclusion of PVDs and in absence of PVDs might (case ۳(a) + case ۳(b) or case ۴(a) + case ۴(b)) might be considered in the case of constant vacuum pressure for preliminary prediction or empirical equations related to lateral displacement.

One of the main obstacles in any FEM modeling simulations or comprising empirical or analytical solutions for combined vacuum and surcharge preloading is excess pore pressure. In the previous sections some aspects of this matter have been explained. Figure ۳(c) shows that for the field variable applied vacuum the resultant curve for excess pore pressure in the SUM case is overestimated while as shown in figure ۴(c) the resultant curve for excess pore pressure till the day ۹۰ is underestimated while for the rest till ۱۶۰th day it is overestimated. Since the applied vacuum pressure was reduced to -۲۰ kpa on the ۱۲۰th day none of the cases could predict the values of excess pore pressure. Figure ۳(c) and ۴(c) show the complicated mechanism of coupling in the dissipation of excess pore pressure. For the ideal case as shown in figure ۵(c) and ۶(c), the SUM curve in some areas overestimates and in some areas underestimates the FEM excess pore pressure because of the coupling effect. The cases assuming PVDs inclusion along with surcharge (case ۳(a) and case ۴(a)) agree best with both the ideal and verified FEM curves and they can be used properly for an acceptable estimation of excess pore pressure in systems with constant and variable vacuum pressure. This agreement by the FEM model is exactly the mechanism that was described in the first part of this literature. Although the vacuum effect is the same as surcharge loading in accelerating the process of consolidation, its acting mechanism is completely different and negative excess pore pressure attributed to vacuum preloading is only a term for describing the magnitude of applied suction through PVDs.

Figure ۳(a) shows a draw-back in the settlement curve on the ۷۵th day in the case ۱(b). This is the time when because of technical problems the applied vacuum pressure has dropped in real-world projects and an unloading condition occurred. In the absence of the surcharge preloading the acquired settlement reduced from ۶۰ cm to ۴۰ cm. As (J.-C. Chai, Carter, & Hayashi, ۲۰۰۶) reported this might be attributed to k_v (no strain condition) where the vacuum pressure is no more larger than k_v condition to maintain the vertical deformation. If even the potential inward forces of the vacuum preloading are neglected, as it can be seen without any surcharge preloading the occurrence of undesirable heave is expected after removal or reduction in the vacuum preloading. This case clearly illuminates the necessity of applying the combined system of the surcharge and vacuum preloading to maintain the efficiency of the whole treatment process. At least ۳۰ percent of preliminary designed preload is recommended for the surcharge preloading.

The false idea might exist that the vacuum preloading necessarily induces surficial inward displacement. In fact as stated by (Jinchun Chai et al., ۲۰۱۳) outward, inward or inward near the ground and outward at greater depth might occur. It can be seen that for all the cases in the verified and ideal cases, outward displacement is dominant except for the case that with PVDs and constant vacuum without surcharge preloading (case ۳(b)) that inward displacement near the ground and outward displacement at greater depth dominates. As (Liu et al., ۲۰۱۹) reported The ground settlement of the clayey soils during vacuum removal is mainly attributed to fact that the Young's modulus in the vertical direction is higher than that in the horizontal direction because of the soil anisotropy, and Lateral displacement is dominant for the ground deformation during vacuum removal. As it can be seen in fig ۳(b), because of the variable applied vacuum pressure the lateral displacement on ground surface is outward similar to the surcharge preloading. If the magnitude and duration of the vacuum pressure don't be high enough to counter-effect the soil anisotropy and k_v state, inward lateral displacement

effect on ground surface should not be expected. This case clearly shows the necessity of constant application of a minimum quantity vacuum pressure that should be maintained the whole time, even if a stepped vacuum pressure is determined in the design procedure.

۵. Conclusion

The following conclusions are based on data, analyses, and interpretation presented in this paper:

۱. Superposition law is not valid in the combined vacuum and surcharge preloading and other phenomena exists which are the interaction between PVDs and vacuum and surcharge or the coupling. The mentioned hydro-mechanical coupling effect can be decreasing or increasing, based on the characteristics of any project.
۲. u_{vs} is the coefficient of consolidation for a combined surcharge and vacuum preloading that considers the effect of coupling in analytical solution and should be accounted for in analytical equations and also in tests which are under combined surcharge and vacuum preloading.
۳. By applying constant vacuum pressure during the predicted time the coupling effect has been minimized in settlement curves.
۴. For discretizing of complex models like combined surcharge and vacuum for settlement prediction, the SUM models that include PVDs with surcharge gives better predictions for both cases with constant and variable applied vacuum pressure although for variable vacuum the results would be underestimated by ۴۰ percent in the final settlement curve.
۵. For cases that include variable vacuum pressure, the lateral displacement prediction can be drawn from the case surcharge without PVDs for empirical equations except for ground surface where ۵۰ percent of SUM in both cases of surcharge with and without PVDs might be considered.
۶. The lateral displacement prediction can be drawn from both models with and without PVDs in the case of constant vacuum pressure for empirical equations.
۷. The cases assuming PVDs inclusion along with surcharge agree best with both the ideal and verified FEM curves and they can be used properly for estimation of excess pore pressure in systems with constant and variable vacuum pressure.
۸. Although the effect of vacuum preloading is somehow the same as surcharge preloading in acceleration of the consolidation process, they shouldn't be mistaken with each other as they have two different mechanisms.
۹. There is a difference in the magnitude and the rate of settlement, lateral displacement, and pore pressure resulting from a vacuum load or an equivalent fill load in combined systems, and as a result of coupling, and different acting mechanisms their effect cannot be used interchangeably.
۱۰. To keep the efficiency of combined vacuum and surcharge preloading the minimum ۳۰ percentage of designed preload is recommended for the surcharge preloading.
۱۱. If the magnitude and duration of the vacuum pressure don't be high enough to counter-effect the soil anisotropy and k_v state, the desired inward lateral displacement on ground surface from vacuum preloading should not be expected. Constant application of a minimum vacuum pressure should be maintained the whole time even if a stepped vacuum pressure is determined in the design procedure.

۶. Acknowledgement:

Some or all data, models, or code that support the findings of this study are available from the corresponding author upon reasonable request. The authors declare there is no competing interest

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